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UNIFIED COUNCIL OLYMPIADS

UIMO
Unified International Mathematics Olympiad

# UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD

***CLASS - 8***

***Question Paper Code : 4O109***

## KEY

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| C | C | A | D | D | B | C | B | C | A |
| **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
| A | C | B | B | A | B | C | D | C | A |
| **21** | **22** | **23** | **24** | **25** | **26** | **27** | **28** | **29** | **30** |
| A | D | B | C | A | D | B | C | A | B |
| **31** | **32** | **33** | **34** | **35** | **36** | **37** | **38** | **39** | **40** |
| A,B,C,D | A,B,C,D | A,C,D | A,B,D | A,B,D | A | B | A | C | B |
| **41** | **42** | **43** | **44** | **45** | **46** | **47** | **48** | **49** | **50** |
| A | C | C | D | A | C | C | A | A | B |

## SOLUTIONS

**MATHEMATICS - 1**

01. (C) We have, $\sqrt{1+\frac{y}{144}}=\frac{13}{12}$

Squaring both sides, we get

$1+\frac{y}{144}=\frac{169}{144}$

$\Rightarrow \frac{y}{144}=\frac{169}{144}-1=\frac{169-144}{144}$

$\Rightarrow \frac{y}{144}=\frac{25}{144} \Rightarrow y=25 \quad \therefore \sqrt{y}=\sqrt{25}=5$

02. (C)

| | | |
|---|---|---|
| 1 | $\overline{3}\ \overline{65}\ \overline{62}$ | 191 |
| | 1 | |
| 29 | 265 | |
| | 261 | |
| 381 | 462 | |
| | 381 | |
| | 81 | |

Number of soldiers left = 81

03. (A) $(x-3)(x^2+3x+9) = x(x^2+3x+9) - 3(x^2+3x+9) = x^3+3x^2+9x-3x^2-9x-27$

$= (x^3 - 27)$

04. (D) Given $\angle A + \angle B + \angle C + \angle D = 182^\circ + 150^\circ + 100^\circ$

$\therefore$ $(\angle A + \angle B + \angle C + \angle D) + \angle B = 432^\circ$

$360^\circ + \angle B = 432^\circ$

$\angle B = 432^\circ - 360^\circ = 72^\circ$

But $\angle A + 72^\circ = 182^\circ \Rightarrow \angle A = 110^\circ$

$72^\circ + \angle C = 150^\circ$

$\angle C = 78^\circ$

$\therefore$ Required sum = $\angle A + \angle B = 182^\circ$

05. (D) Let $q_1$, $q_2$, $q_3$ and $q_4$ be the four required rational numbers. Then,

$$q_1 = \frac{1}{2}\left(\frac{1}{6}+\frac{1}{3}\right) = \frac{1}{2}\left(\frac{1+2}{6}\right) = \frac{1}{4}$$

$$q_2 = \frac{1}{2}\left(\frac{1}{4}+\frac{1}{3}\right) = \frac{1}{2}\left(\frac{3+4}{12}\right) = \frac{7}{24}$$

$$q_3 = \frac{1}{2}\left(\frac{1}{4}+\frac{7}{24}\right)$$

$$= \frac{1}{2}\left(\frac{6+7}{24}\right) = \frac{1}{2}\left(\frac{13}{24}\right) = \frac{13}{48}$$

and $q_4 = \frac{1}{2}\left(\frac{7}{24}+\frac{13}{48}\right)$

$$= \frac{1}{2}\left(\frac{14+13}{48}\right)$$

$$= \frac{1}{2}\left(\frac{27}{48}\right) = \frac{27}{96}$$

$\therefore \frac{1}{4}, \frac{7}{24}, \frac{13}{48}$ and $\frac{27}{96}$ are the required rational numbers between $\frac{1}{6}$ and $\frac{1}{3}$.

06. (B) Let the required number be $x$

According to the question

$$(-24)^{-1} \div x = \left(\frac{4}{9}\right)^{-1}$$

$$\Rightarrow \frac{(-24)^{-1}}{x} = \left(\frac{4}{9}\right)^{-1}$$

$$\Rightarrow x = (-24)^{-1} \div \left(\frac{4}{9}\right)^{-1}$$

$$\Rightarrow x = \frac{1}{-24} \div \left(\frac{9}{4}\right)$$

$$\Rightarrow x = \frac{-1}{24} \times \frac{4}{9} = \frac{-1}{54}$$

07. (C) Let the three consecutive numbers be $x, x+1, x+2$

Given $2x + 3(x+1) + 4(x+2) = 182$

$2x + 3x + 3 + 4x + 8 = 182$

$9x = 182 - 11 = 171$

$$x = \frac{171}{9} = 19$$

08. (B) Number of numbers less than 6 from numbers 1 to 10 is 5

$\Rightarrow$ The probability of getting a number less than 6 is $\frac{5}{10} = \frac{1}{2}$

09. (C) $y^2 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3$

$= 100 + 125 = 225$

$y^2 = 15^2$

$\therefore$ $y = 15$

10. (A) The sum $1 + 3 + 5 + 7 + \ldots\ldots + 29$ is an arithmetic sequence with the first term 1, common difference 2 and last term 29. The sum of sequence is calculated using the formula $S_n = \frac{n}{2} \times (a + L)$

$S_{15} = \frac{15}{2} \times (1 + 29)$

$= \frac{15}{2} \times 30 = 15 \times 15 = 225$

11. (A) $$\frac{y}{60} = \frac{3}{2} - \frac{1}{6} - \frac{7}{15} - \frac{3}{4}$$

$$\frac{y}{60} = \frac{90 - 10 - 28 - 45}{60}$$

$$\frac{y}{60} = \frac{7}{60}$$

$y = 7$

12. (C) LHS = $4^2 + 5^2 + ... + 11^2 + 12^2$

$= (1^2 + 2^2 + 3^2 + 4^2 ... + 12^2) - (1^2 + 2^2 + 3^2)$

$= 650 - 14 = 636$

13. (B) Let the numbers be $4x$, $3x$ and $2x$.
Then, $(4x)^3 + (3x)^3 + (2x)^3 = 21384$

$\Rightarrow$ $64x^3 + 27x^3 + 8x^3 = 21384 \Rightarrow 99x^3 = 21384$

$\Rightarrow$ $x^3 = \frac{21384}{99} = 216 \Rightarrow x^3 = 6 \times 6 \times 6$

$\Rightarrow$ $x = \sqrt[3]{6 \times 6 \times 6} = 6$

Hence, the numbers are $2x = 2 \times 6 = 12$,

$3x = 3 \times 6 = 18$ and $4x = 4 \times 6 = 24$

$\therefore$ Required sum = 12 + 18 + 24 = 54

14. (B) $\text{LHS} = \sqrt[3]{\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times .... \times \frac{999}{1000}}$

$= \sqrt[3]{\frac{1}{1000}} = \frac{1}{10}$

15. (A) Let the provisions last for $x$ days.

| | | |
|---|---|---|
| **Number of men** | 800 | 800 + 200 = 1000 |
| **Number of days** | 60 | $x$ |

More the number of men, less will be the number of days, provisions last. It is a case of inverse proportion.

$\therefore$ 800 : 1000 = $x$ : 60

$\Rightarrow$ $\frac{800}{1000} + \frac{x}{60}$

$\Rightarrow$ $x = \frac{800 \times 60}{1000} = 48$

Hence, the provision will last for 48 days.

16. (B) $\text{CI} - \text{SI} = \text{P}\left(\frac{\text{r}}{100}\right)^2$

$\text{Rs. } 96 = \text{Rs. } 15000\left(\frac{\text{r}}{100}\right)^2$

$\text{Rs. } 96 \times \frac{2}{3} = \text{r}^2$

$r^2 = 64 = 8^2$

r = 8

17. (C) Given, P = Rs. 8000, r= 15%

and n = 3 years

$\therefore$ $A = P\left(1 + \frac{r}{100}\right)^n = 8000\left(1 + \frac{15}{100}\right)^n$

$= 8000 \times \left(\frac{115}{100}\right)^3 = 8000 \times \left(\frac{23}{20}\right)^3$

= Rs. 12167

$\Rightarrow$ C.I. = A – P = Rs. (12167 – 8000)

= Rs. 4167

18. (D) WX // YZ and $\angle XWY = \angle WYZ = 35^\circ$

$\angle XYW = 180^\circ - 108^\circ - 35^\circ$

$= 180^\circ - 143^\circ = 37^\circ$

19. (C) $= x^{\frac{a+b-c}{(a-c)(b-c)} + \frac{b+c-a}{(b-a)(c-a)} + \frac{c+a-b}{(c-b)(a-b)}}$

$= x^{\frac{(a-b)(c-a-b)+(b-c)(a-b-c)+(c-a)(b-c-a)}{(a-b)(b-c)(c-a)}}$

$= x^{\frac{ac-bc-a^2+ab-ab+b^2+ab-ac-b^2 \\ +bc-bc+c^2+bc-ab-c^2+ac-ac+a^2}{(a-b)(b-c)(c-a)}}$

$= x^0 = 1$

20. (A) $4^x\left[1 - \frac{1}{4}\right] = 24$

$\Rightarrow$ $4^x = \frac{24^{8} \times 4}{3}$

$\Rightarrow$ $2^{2x} = 32 = 2^5$

$\therefore$ $x = \frac{5}{2}$

$\therefore$ $(2x)^x = \left(2 \times \frac{5}{2}\right)^{5/2} = 5^{5/2}$

21. (A) $\left(x^2-\frac{1}{x^2}\right)\left(x+\frac{1}{x}\right)=x^3+\frac{x^2}{x}-\frac{x}{x^2}-\frac{1}{x^3}$

$=\left(x^3+x-\frac{1}{x}-\frac{1}{x^3}\right)$

22. (D) $\frac{p\left(1+p+p^2+\ldots p^6\right)}{p^{-9}\left(1+p+p^2+\ldots p^6\right)}=p^{10}$

23. (B) $(a+b)^3=a^3+3a^2b+3ab^2+b^3$

$(63+37)^3=63^3+3(63)^2(37)+3(63)(37)^2+(37)^3$

$\therefore$ $(63+37)^3=(100)^3=100\times100\times100$

$=1000000$

24. (C) $\frac{(67.542)^2-(32.458)^2}{100}$

$=\frac{100\times35.084}{100}=35.084$

25. (A) TSA of a cylinder = CSA of cylinder + 2 × base area

$=2\pi rh+2\pi r^2$

$=143\ cm^2+2\times38.5\ cm^2$

$=143\ cm^2+77\ cm^2$

$=220\ cm^2$

26. (D) $(1-x)(1+x+x^2)+(1+x)(1-x+x^2)$

$=(1-x^3)+(1+x^3)=2$

27. (B) Since, $6^{2y}\div6^{-4}=6^{24}$

$\Rightarrow$ $6^{2y-(-4)}=6^{24}$

$\Rightarrow$ $6^{2y+4}=6^{24}$

28. (C) $(-20)+(-15)+\frac{(-20)\times(-15)}{100}=-32$

29 (A) $\frac{-11}{12}=-0.91$

$\frac{-12}{13}=-0.92$

$\frac{-14}{15}=-0.93$

$\frac{-20}{21}=-0.95$

$-0.91>-0.92>-0.93>-0.95$

$\therefore$ $\frac{-11}{12}$ is the greatest.

30. (B) Given $2\pi r(h+r)=8800\ cm^2$

$2\times\frac{22}{7}r(5+r)=8800$

$5r+r^2=8800\times\frac{7}{44}$

$r^2+5r-1400=0$

$r^2+40r-35r-1400=0$

$r(r+40)-35(r+40)=0$

$(r+40)(r-35)=0$

$r+40=0$ (or) $r-35=0$

r= –40 is rejected because length is never negative.

$\therefore$ r = 35 cm

## MATHEMATICS - 2

31. (A,B,C,D)

$25^2-7^2=625-49=576=24^2$

$17^2-8^2=289-64=225=15^2$

$12^2+5^2=144+25=169=13^2$

32. (A, B, C, D)

Given $x_1y_1=x_2y_2=x_3y_3=x_4y_4$

$4\times6=24=3\times8=12\times2=6\times4=5\times4.8$

33. (A,C,D)

$2x^2-x-6=2x^2-4x+3x-6$

$=2x(x-2)+3(x-2)$

$=(x-2)(2x+3)$

$3x^2-8x-4$ can n't be factorised.

$3x^2-8x+4=3x^2-6x-2x+4$

$=3x(x-2)-2(x-2)$

$=(x-2)(3x-2)$

$5x^2-18x+16=5x^2-10x-8x+16$

$=5x(x-2)-8(x-2)$

$=(x-2)(5x-8)$

34. (A, B, D)

Total ratio of 9 + 1 = 10

Total ratio of 7 + 3 = 10

Total ratio of 1 + 10 = 11 ≠ 10

Total ratio of 1 + 4 = 5 = 2(5) = 10

35. (A, B, D)

$$2^{-6} = \frac{1}{2^6} = \frac{1}{64}$$

$$6^{-2} = \frac{1}{6^2} = \frac{1}{36}$$

$$\frac{1}{64} < \frac{1}{36} \Rightarrow 2^{-6} < 6^{-6}$$

$2^{10} = 1024$

$10^2 = 100$

$\therefore$ $2^{10} > 10^2$

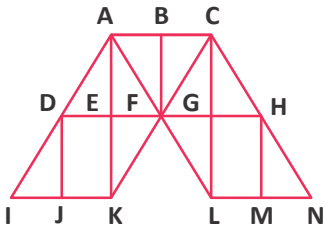
## REASONING

36. (A) From the observation of die thrown 4 times.

37. (B) 

38. (A) Buy some thing means "PURCHASE"

39. (C) ABF, CBF, AFE, EFK, AED, DIJ, FLG, CFG, CGH, HNM, AFG, AFK, AIK, CFL, CLN, ALC, AKC




40. (B) (8×5×4) – (8 + 5 + 4) = 160 – 17 = 143

(3×7×4) – (3 + 7 + 4) = 84 – 14 = 70

(2×3×6) – (2 + 3 + 6) = 36 – 11 = 25

(2×1×5) – (2 + 1 + 5) = 10 – 8 = 2

41. (A) On writing the first half of the English alphabet in reverse order:

M L K J I H G F Ⓔ D C B A N O P Q R S T U V W X Y Z

NOW 18 – 9 = 9 (left to left)

9th letter from the left is E.

42. (C) From the code language; codes for respective words are given below:

Manchi → Fine

Kapda → Cloth

Saaf → Clear

Neeru → Water

Havaman → Weather

Hence code for water is neeru.

Option (C) is correct answer.

43. (C) The sequence alternates between boxes having one or two shapes.

The lines within each box alternates from being parallel to being adjacent. The missing box must have two shapes and two parallel lines.

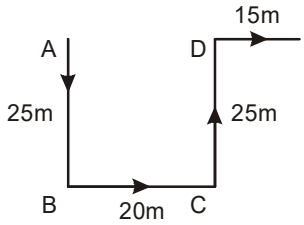
44. (D) Number of parallelograms of one component = 4

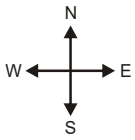
Number of parallelograms of two components = 3

$\therefore$ Total number of parallelograms = 7

45. (A) Requires distance = AE = AD + DE

= 20 + 15 = 35m



## CRITICAL THINKING

46. (C) Let us solve it backwards:

Two days after Monday=Wednesday

Immediately after Wednesday = Thursday

Two days after Thursday=Saturday

Three days after Saturday=Tuesday

47. (C) Rohit is 3rd.

Raj is behind Rohit and there are two people between Raj and Rohit. Therefore Raj is 6th.

There is one person between Mark and Raj. So Mark can be either 4th or 8th. There are 4 people between Rohit who is 3rd and Mark. Therefore between 4th and 8th, Mark can only be 8th.

Now Mark is 8th from beginning and 7th from end which means there are 6 people behind Mark. Therefore there are a total of 8+6=14 people in the queue. Hence option C is the answer.

48. (A) 10 Rabbits = 2 goats

9 goats = 3 cows

8 cows = 2 horses

5 horses = ? Rabbits

1 horse = 4 cows

5 horses = 4 × 5 cows = 20 cons

5 horses = 20 × 3 goats

5 horses = 60 goats

5 horses = 60 × 5 Rabbits

5 horses = 300 Rabbits

49. (A) AGREEMENT

50. (B) false

According to the first two statements - C > D > A > B

So, it is clear that C is larger among others. So, the third statement is false as it is stating that B is larger.

*The End*